Discontinuous stationary solutions to reaction-diffusion-ODE systems

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I will consider system of n ordinary differential equations coupled with a single reaction-diffusion equation in an open and bounded domain $\Omega \subseteq \mathbb{R}^N$

$$\begin{aligned} \boldsymbol{u}_t &= \boldsymbol{f}(\boldsymbol{u}, v), \quad x \in \overline{\Omega}, \quad t > 0, \\ v_t &= g(\boldsymbol{u}, v), \quad x \in \Omega, \quad t > 0, \end{aligned} \tag{1}$$

where

$$\boldsymbol{u} = \boldsymbol{u}(x,t) = \begin{pmatrix} u_1(x,t) \\ \vdots \\ u_n(x,t) \end{pmatrix} \quad \text{and} \quad v = v(x,t). \tag{2}$$

Our goal is to identify class of stationary solutions and examine their stability. System (1) may have different types of stationary solutions

1. Regular stationary solutions
2. Jump-discontinuous stationary solutions

I will present a construction of discontinuous stationary solutions to general reaction-diffusion-ODE systems and show sufficient conditions for their stability.