The Non-Monotone Waves in the Two-Species Lotka-Volterra Competition Model and Application

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Abstract

We investigate the Lotka–Volterra competitive reaction-diffusion equation with a focus on the co-existence phenomenon. To understand the system's dynamics, we investigate traveling wave solutions.

Our goal is to find solutions connecting the two equilibria $(0,0)$ and (u^*, v^*). We consider the following system

$$\begin{cases} u'' - su' + u(1-u-cv) = 0, \ \xi \in \mathbb{R}, \\ dv'' - sv' + v(a-bu-v) = 0, \\ (u,v)(-\infty) = (0,0), \ (u,v)(+\infty) = (u^*, v^*), \end{cases} \tag{1}$$

under the weak competition condition;

$$b < a < \frac{1}{c}.$$

Previous studies have established the existence of strictly monotone solution; however, the solution is not unique.

In this work, we successfully construct the non-monotone solution. We also use this solution to construct the front-pulse solution.